Community Structure in Complex Networks

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April 27, 2018

Outline

- Introduction
- Hierarchical community structure
- Structural hole spanners
- Overlapping community structure
- Community search
- Conclusion

The problems:

- Hierarchical community detection
- Identifying top-k structural hole spanners
- Overlapping community detection
- Community search

The thesis

- Formalises these problems, and
- Proposes efficient algorithms for them, and
- Presents extensive experimental results on real datasets

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Complex networks

Appear in a wide range of contexts,

- Biology
- Sociology
- Citation networks
- Collaboration networks
- World Wide Web network

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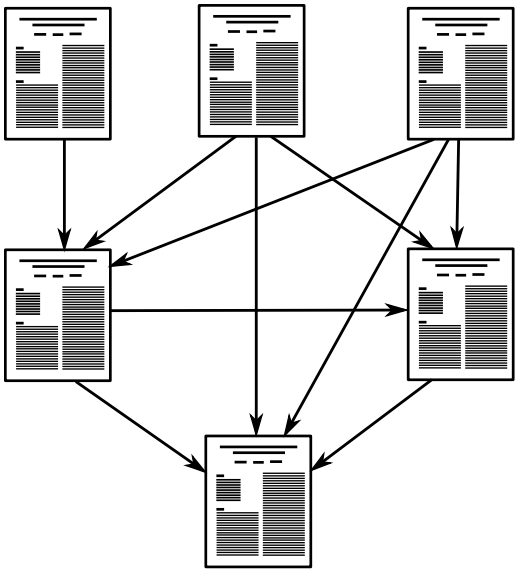
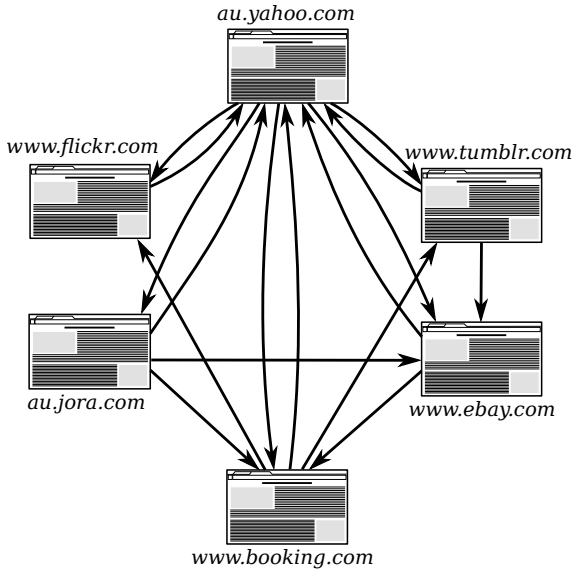
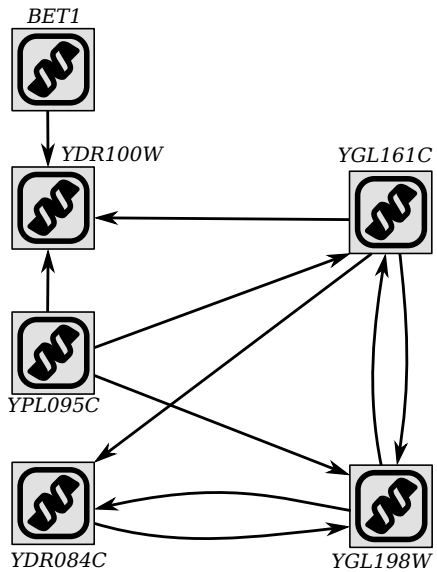
Examples of complex networks



(a) Citation network



(b) Hyperlink network



(c) Protein network

Community structure

Edges appear with high density among groups of vertices, while with less density between groups,

- Social networks
- Network of bloggers
- Collaboration networks
- World Wide Web network

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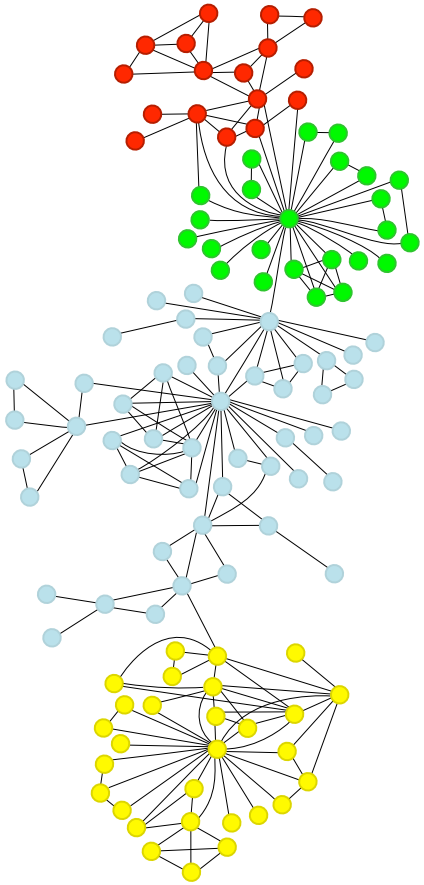
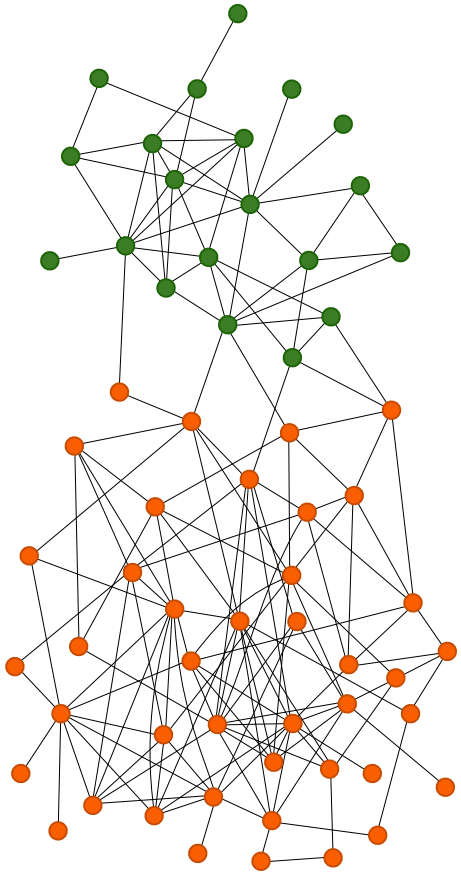
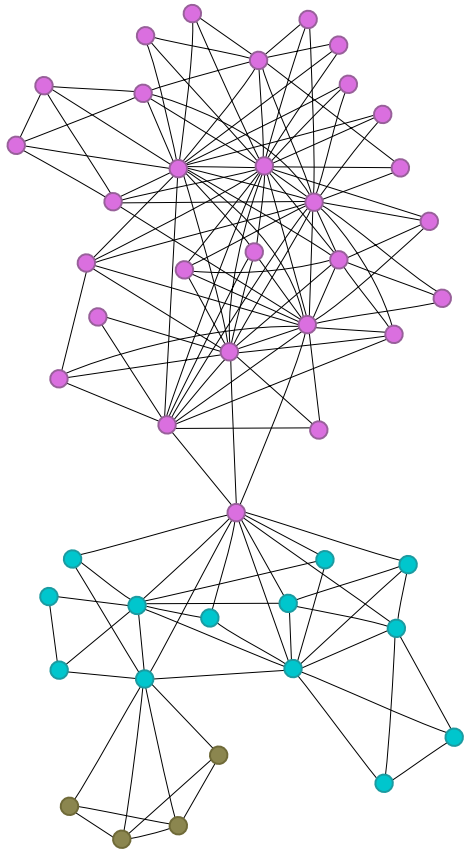
Examples of community structure



(d) Collaboration network



(e) Network of dolphines



(f) PPI network

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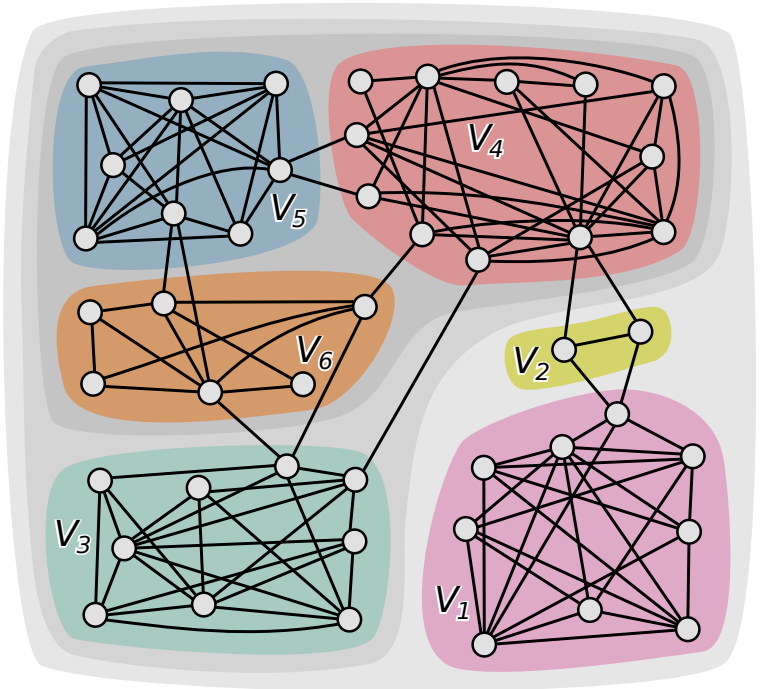
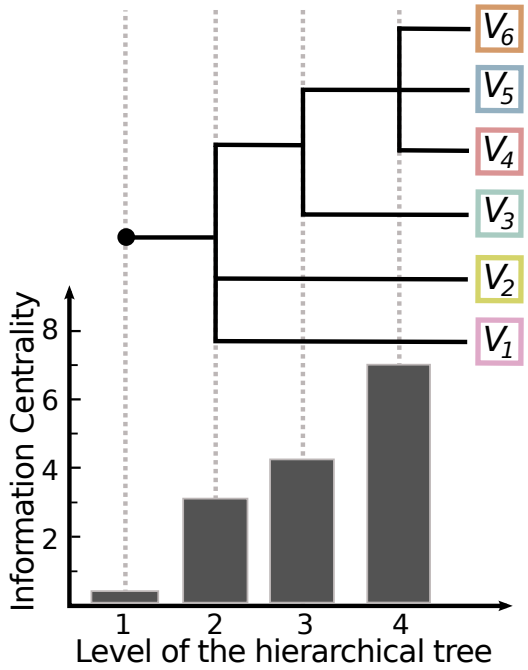
Hierarchies among communities

Communities can be successively partitioned into denser and denser communities.





(*) Information centrality of a subgraph is the inverse of average distance in the subgraph.

Existing approaches

Top-down: Isolate communities by removing edges based on:

- Betweenness centrality of edges
- Impact on the information centrality
- Impact on the modularity
- Edge-cuts, vertex-cuts and triangles

Bottom-up: Expand communities using fitness metrics from:

- Random vertices
- Maximal cliques
- Maximum degree vertices

Major challenges

- Scalability of the algorithms
- Accuracy of the modelling

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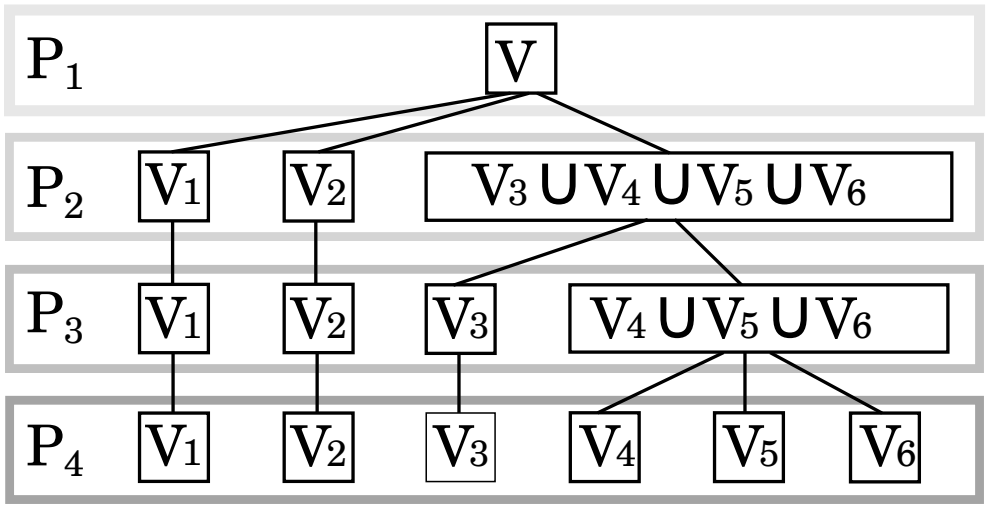
Cohesive hierarchies

Communities in lower levels of the hierarchy are more densely connected to each other.

In a **Cohesive Hierarchy** $\mathcal{P}$, communities at level k are connected with no larger than k edges.



Hierarchical community detection problem

Given a network G, find a cohesive hierarchy $\mathcal{P}$ of communities, where the sum of information centralities of the subgraphs in $\mathcal{P}$ is maximized, i.e.,

$$\text{maximize} \sum_{P_i \in \mathcal{P}} \sum_{V' \in P_i \setminus P_{i-1}} \mathcal{D}(G[V']).$$

Theorem. The hierarchical community detection problem is NP-hard.

Basic algorithm (CHD)

1. Let $P_k = \emptyset$ be the set of communities at level k;
2. Let $P_1 \leftarrow V$, $k \leftarrow 1$;
3. While $(\mathcal{D}(P_k) > \mathcal{D}(P_{k-1}))$
4. $\quad$ Let $k \leftarrow k + 1$;
5. $\quad$ Remove all k-cuts in each $V_i \in P_{k-1}$;
6. Return $\mathcal{P} = \{P_1, \cdots, P_k\}$;

Sparse certificate

We can use a $(k+1)$-sparse certificate of G for finding k-cuts.

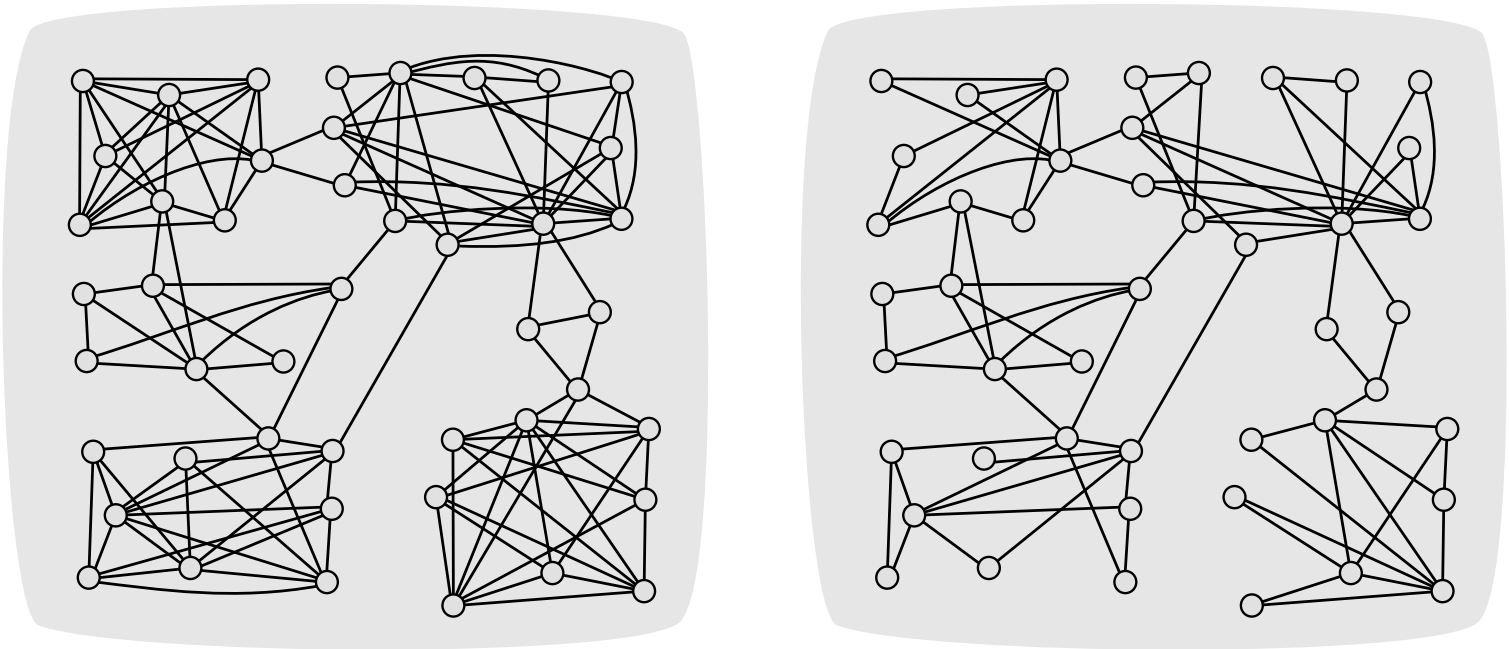


Figure 1 : Graph G and its 2-sparse certificate.

The fast algorithm (FACH)

1. Let $P_k = \emptyset$ be the set of communities at level k;
2. Let $P_1 \leftarrow V$, $k \leftarrow 1$;
3. While $(\mathcal{D}(P_k) > \mathcal{D}(P_{k-1}))$
4. Let $k \leftarrow k + 1$;
5. Find $(k+1)$-sparse certificate of G;
6. Remove all k-cuts in each $V_i \in P_{k-1}$;
7. Return $\mathcal{P} = \{P_1, \cdots, P_k\}$;

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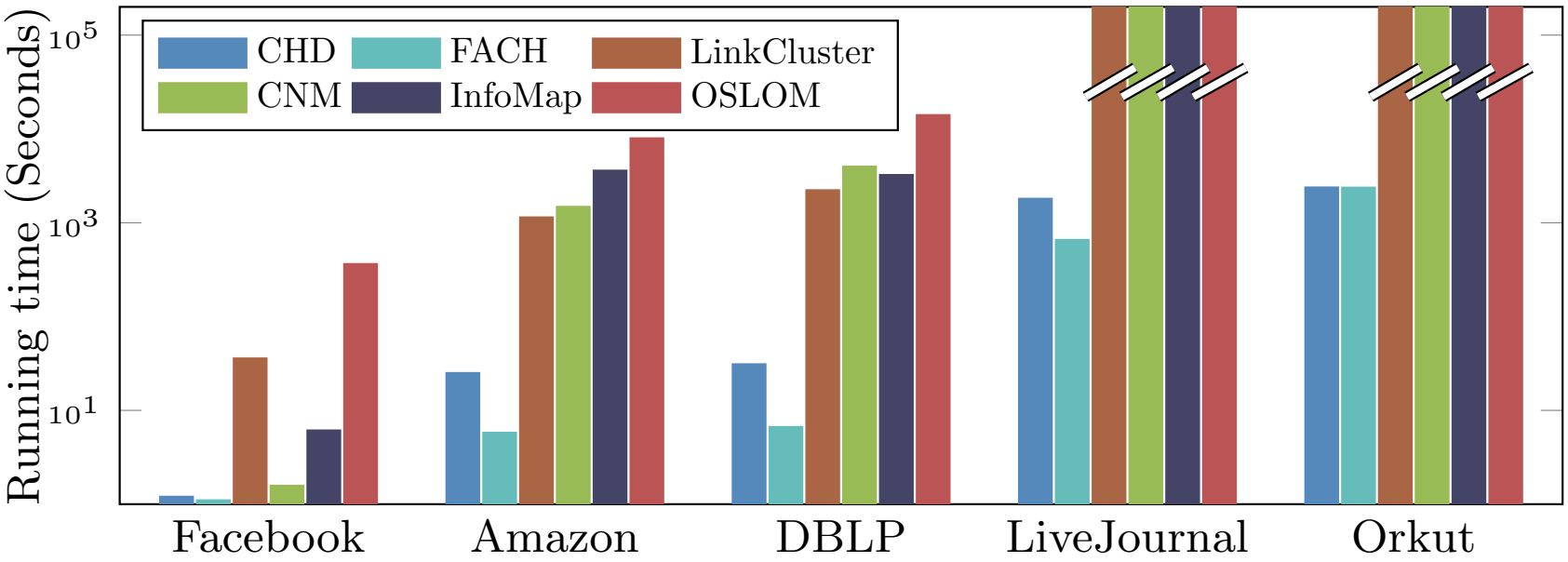
Running Time



CHD and FACH are the proposed algorithms, where CHD does not have the sparsification step and FACH uses the sparsification.

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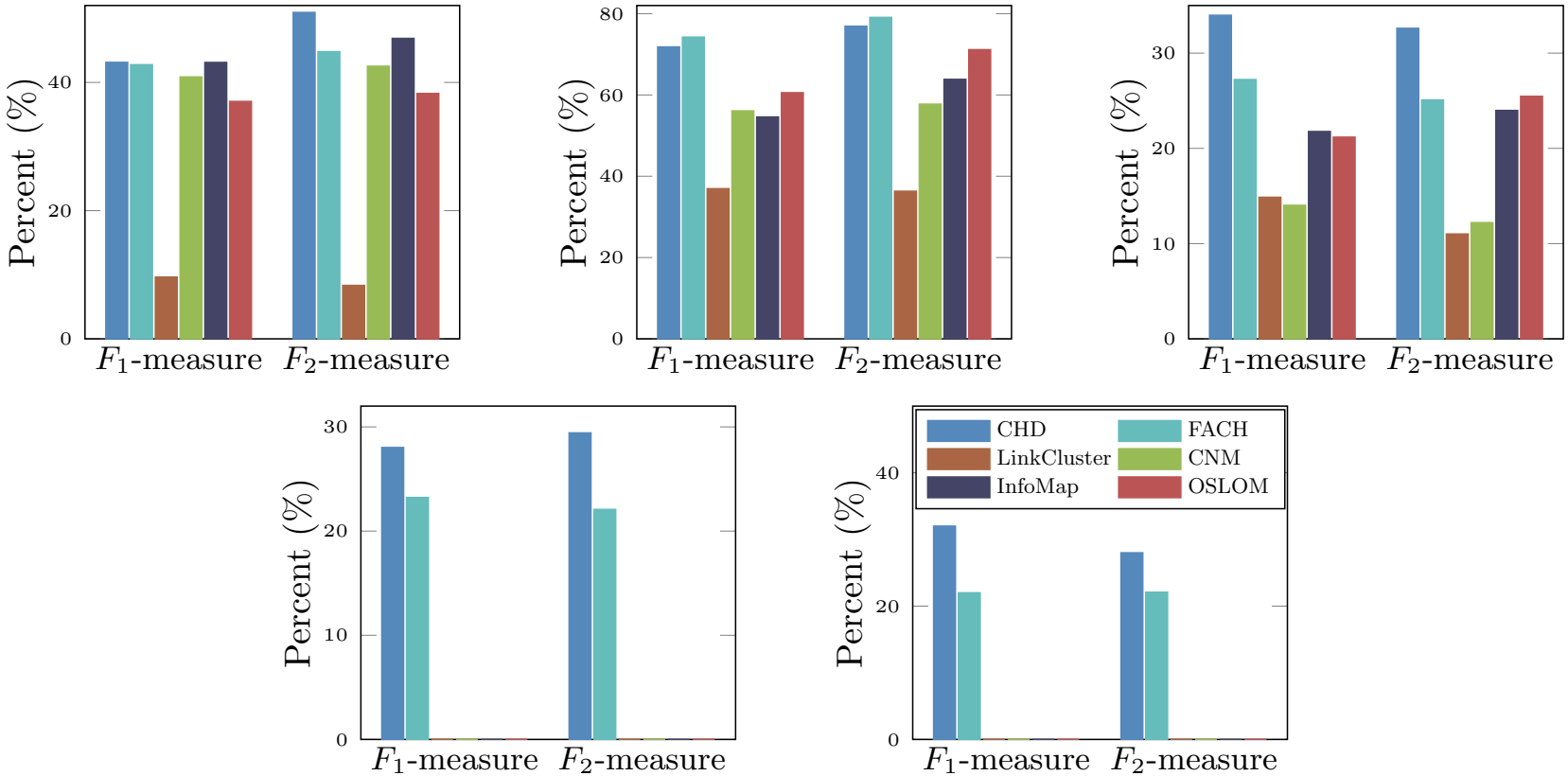
Accuracy



F_1-measure and F_2-measure are calculated by applying a factor α_i to the accuracy of communities at level i.

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Structural holes

Vertices that can bridge diverse groups and bring benefits to the beholder. Having the following properties,

- *Few neighbours*
- *Many communities*
- *Large communities*

Applications of structural hole spanners

Individuals who fill the structural holes, useful for,

- *Community detection*
- *Information spread control*
- *Disease spread control*
- *Strategic network formation*
- *Graph compression*

Existing approaches

Community-based models:

- Ground-truth communities are not available
- The result is sensitive to the found communities
- Definition of a community varies
- Finding communities in a network is difficult

Structural-based models:

- Vertices that reside on more shortest paths
- Vertices that reside on more shortest paths of length 2

Problem definition

The top-k structural hole spanners are a set V_S of k vertices such that their removal will result in the maximum increase on the sum of all pairs of shortest paths, i.e.,

$$\max_{V_S \subset V} \sum_{u,v \in V \setminus V_S} d_{G \setminus V_S}(u, v) \tag{1}$$

Theorem. The problem of identifying top-k structural hole spanners is NP-hard.

Removal of a vertex that is

- within community doesn't increase the distances significantly
- a structural hole spanners greatly changes the distances
- a structural hole spanners that bridge **more communities** changes the distances more considerably
- a structural hole spanner that bridge **larger communities** is more significant in distances

Algorithm based on closeness centrality

$$C(G \setminus \{v\}) = C(G) - \boxed{2\sum_{u \in V} d_{uv}^{G}} + \sum_{u,w \in V} (d_{uw}^{G \setminus \{v\}} - d_{uw}^{G})$$

Algorithm 1 ICC(G,k)

Input: $G = (V, E), k$

Output: k structural hole spanners

1: Calculate $c(v)$ for every vertex $v \in V$;

2: Find k vertices with smallest $c(v)$ from V;

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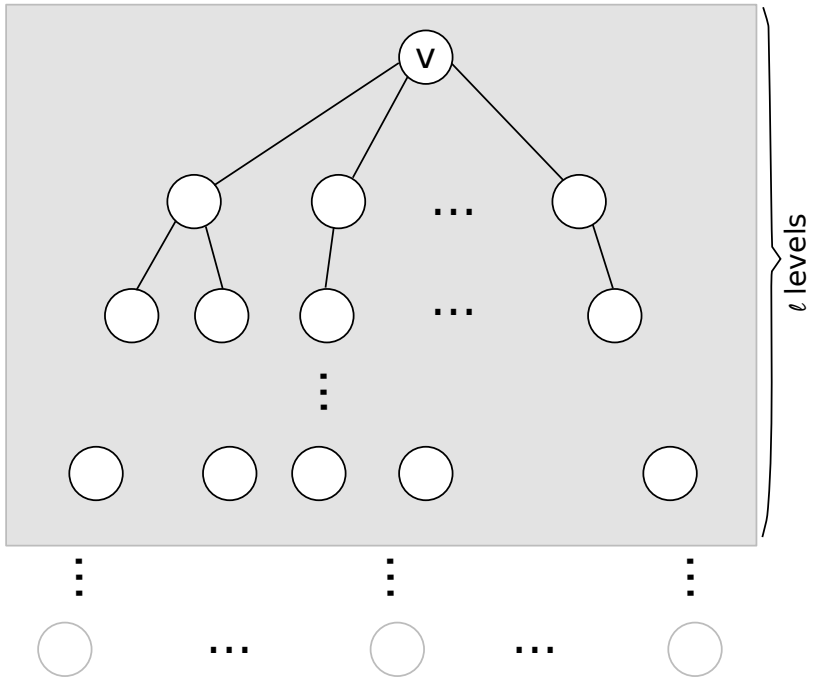
Bounded inverse closeness centrality

$T_\ell(v)$



$$c_\ell(v) = \sum_{u \in T_\ell(v)} d_{uv}^G/(n-1).$$

Bounded inverse closeness centrality

Algorithm 2 BICC(G,k,K,ℓ)

Input: $G = (V, E), k, K, \ell$

Output: k structural hole spanners

1: Calculate $c_\ell(v)$ for every vertex $v \in V$;
2: Find K vertices, H, with **largest** $c_\ell(v)$, from V;
3: Calculate $c(v)$ for every vertex $v \in V$;
4: Find k vertices with smallest $c(v)$ from H;

Algorithm based on articulation points

Having a virtual edge with weight n^3 between every disconnected pair of vertices. Let $c'(v)$ be the sum of weight of virtual edges in the resulting graph after removal of v from G.

Algorithm 3 AP_BICC(G,k,K,ℓ)

Input: $G = (V, E), k, K, \ell$

Output: k structural hole spanners

1: Calculate $c'(v)$ for each vertex $v \in V$;
2: **if** number of articulation points k' is larger than k **then**
3: **output** top-k articulation points with the largest $c'(\cdot)$;
4: **else**
5: **output** k' articulation points and $k - k'$ candidates of BICC.

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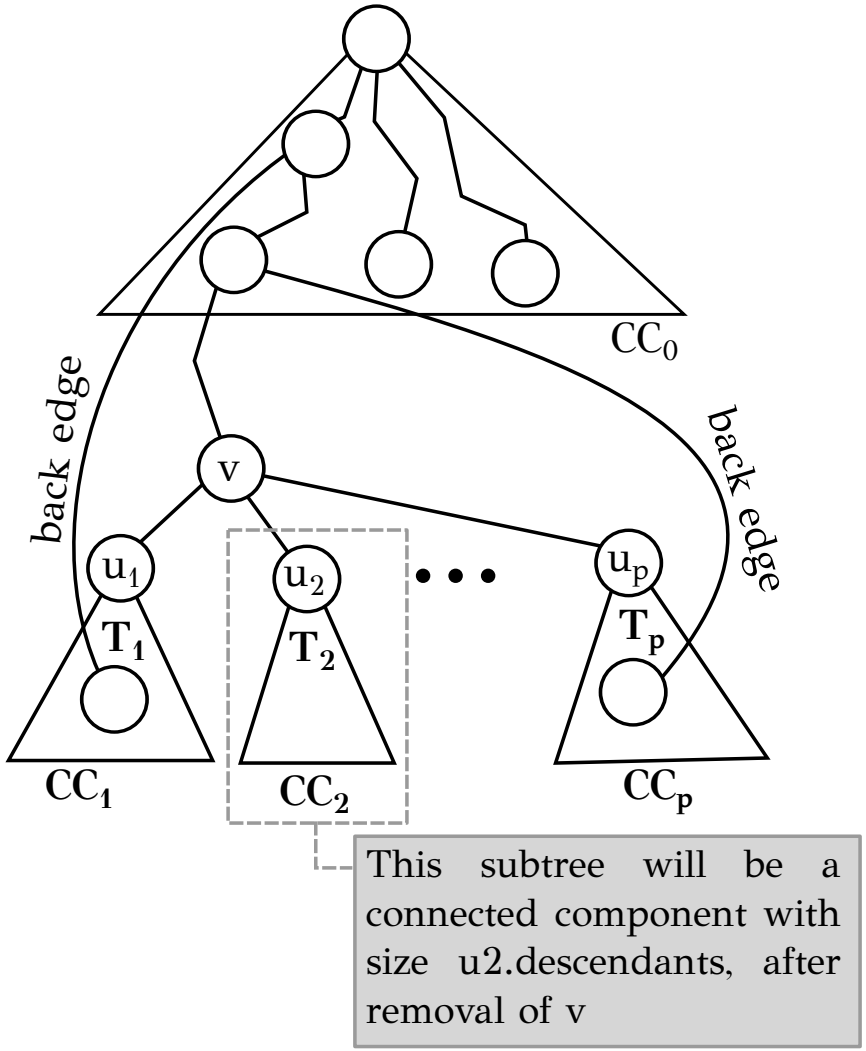
Algorithm based on articulation points

Lemma

Given a graph $G = (V, E)$, the value of $c'(v)$ for all vertices $v \in V$ can be calculated in $O(n + m)$ time.



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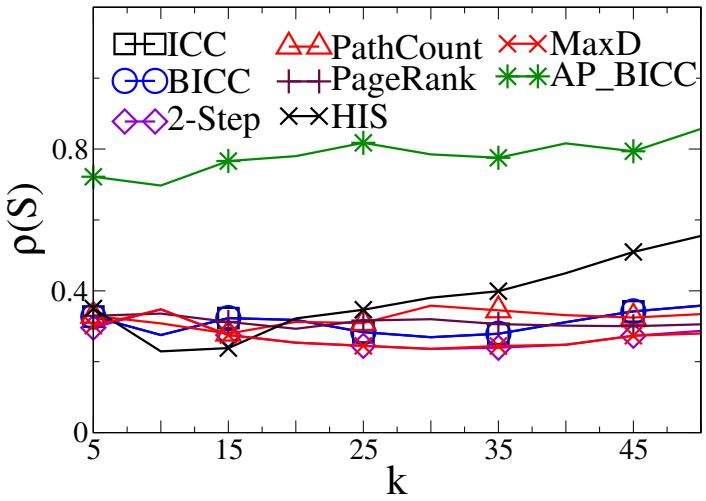
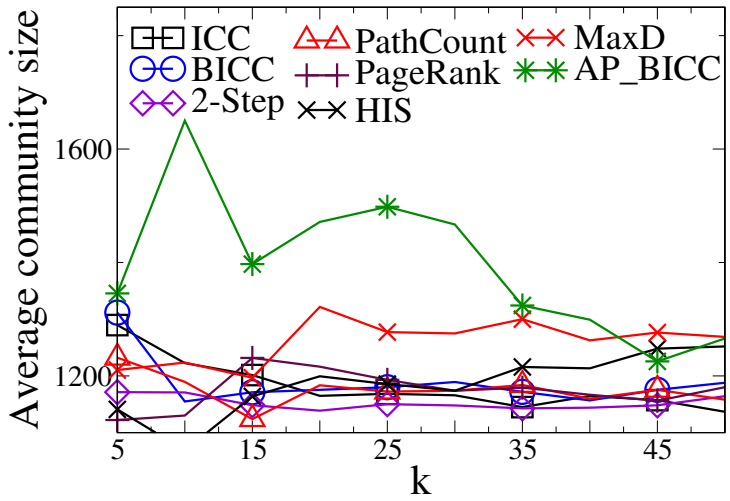
Effectiveness of the model using DBLP



(a) Community membership

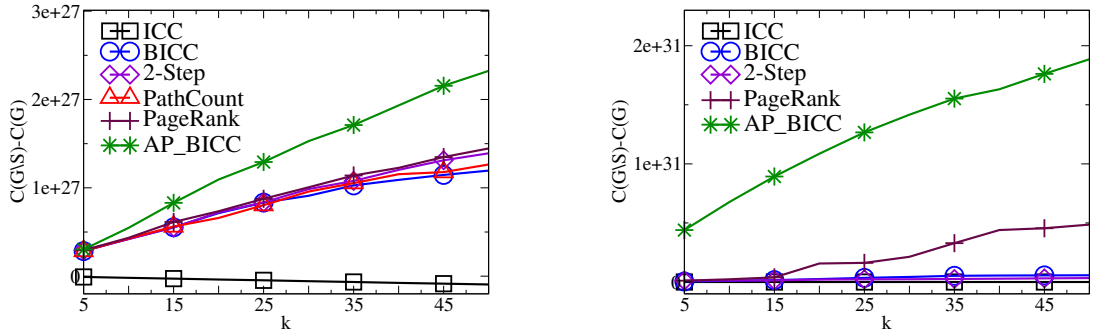


(b) Average community size

Figure 2 : Effectiveness of different algorithms on dataset DBLP using different quality metrics.

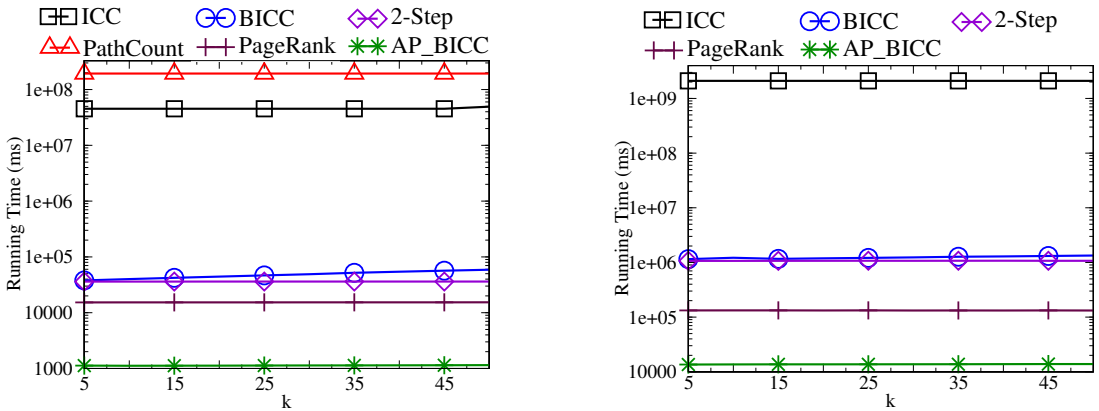
$$\rho(S) = \frac{\sum_{v \in S} (\#\text{ of communities that } v \text{ is connected to})/\deg(v)}{|S|}$$

Performance on real datasets



(a) DBLP-2011 (b) LiveJournal

Figure 3 : Performance of various algorithms, using different datasets.



(a) DBLP-2011 (b) LiveJournal

Figure 4 : Running time of various algorithms, using different datasets.

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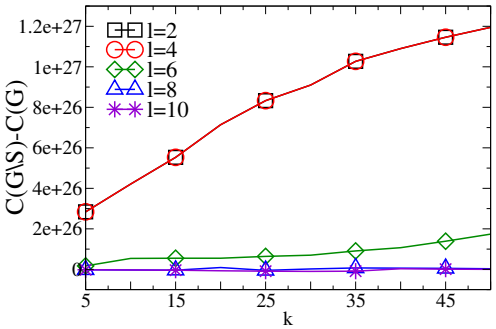
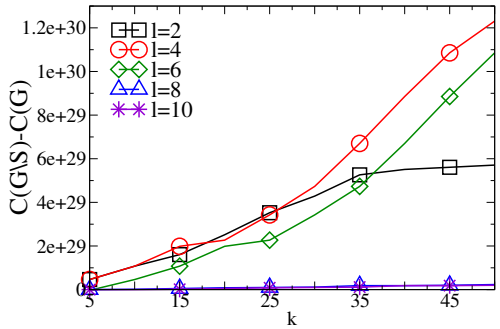
Impact of parameter ℓ on the performance

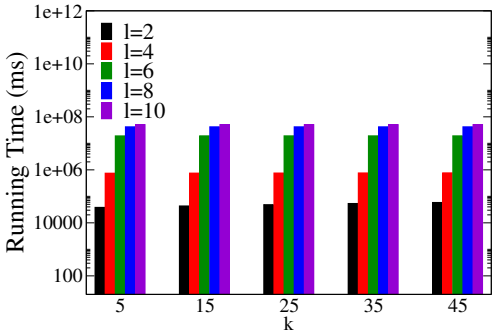


(a) DBLP-2011

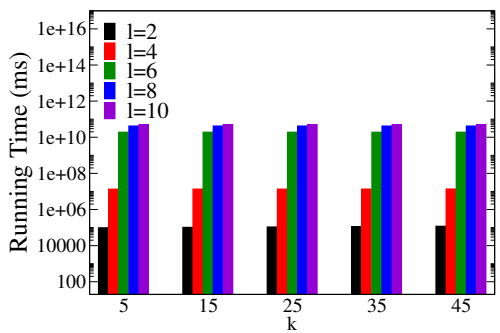


(b) LiveJournal

Figure 5 : Performance of various algorithms, varying parameter ℓ.



(a) DBLP-2011



(b) LiveJournal

Figure 6 : Running time of various algorithms, varying parameter ℓ.

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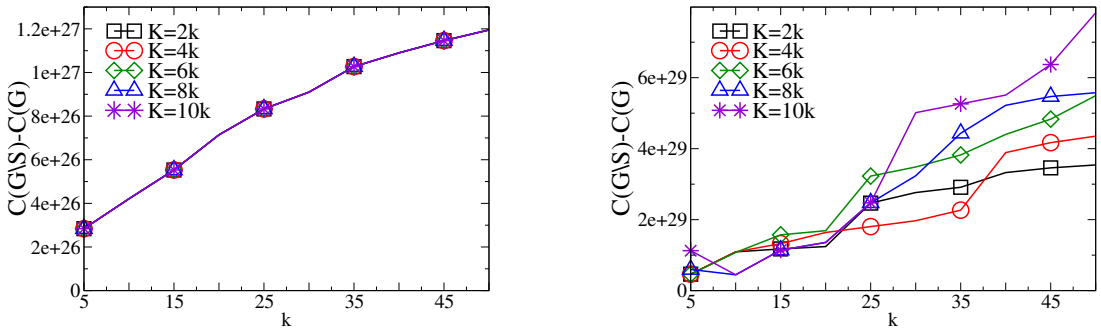
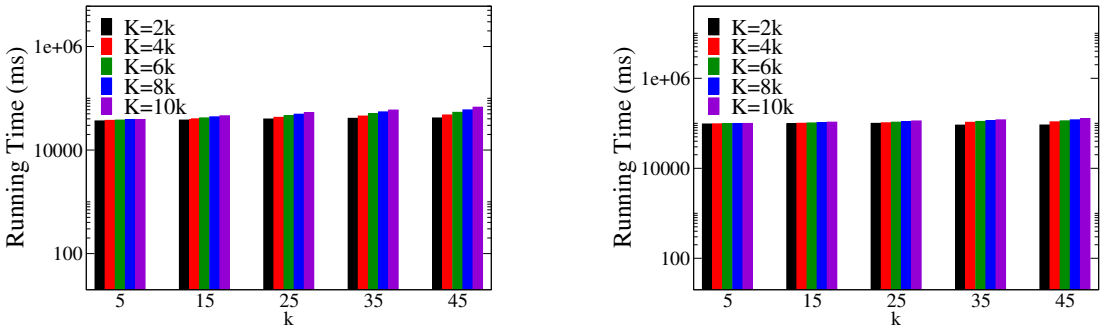
Impact of parameter K on the performance



(a) DBLP-2011 (b) LiveJournal

Figure 7 : Performance of various algorithms, varying parameter K.



(a) DBLP-2011 (b) LiveJournal

Figure 8 : Running time of various algorithms, varying parameter K.

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Overlapping communiy structure

Communities in complex networks overlap with each other:

- Members of social networks join more than 1 communities
- Researchers work in multidisciplinary areas
- Contents of pages may cover more than one topic

Existing approaches

Fuzzy community detection:

- Non-negative matrix factorization
- Non-linear constrained optimization

Agent-based methods:

- Speaker-Listener Propagation Approach (SLPA)
- Demon

Local expansion methods:

- Based on personal page-rank method and fitness metrics
- Clique expansion methods

Free rider and separation effects

- Free rider effect: when there are two communities A & B,

$$f(A) \geq f(A \cup B) \quad \wedge \quad f(B) < f(A \cup B).$$

- Separation effect: when for any two communities A & B,

$$f(A) + f(B) < \max\{f(A) + f(B - A \cap B), f(B) + f(A - A \cap B)\}.$$

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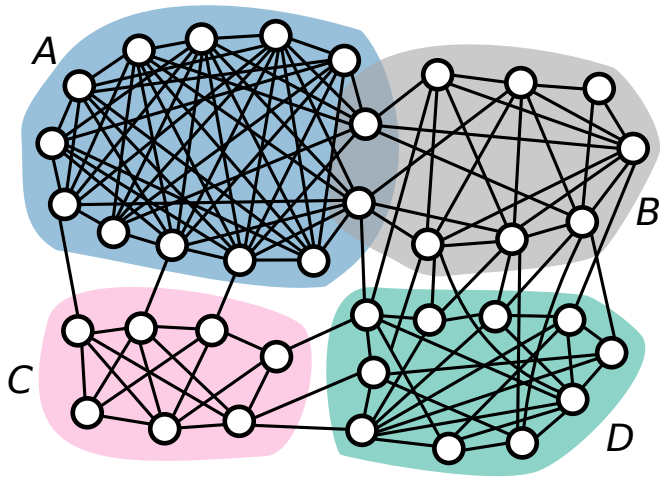
Free rider and separation effects



Fitness metrics	Communities	Remarks
Classic density (CD)	A, $A \cup B$	Free rider effect
Relative density (RD)	A, $A \cup B$	Free rider effect
Subgraph modularity (SM)	A, $A \cup B$	Free rider effect
Local modularity (LM)	B, $A - B$	Separation effect
Conductance (CN)	A, $B - A$	Separation effect

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Overlapping community fitness metric

- More triangles in a community imply a better community
- Community triangles: all its vertices belong to one community
- Fewer triangles between communities imply cohesiveness
- Cut triangle: two of its vertices are in one community, and one vertex is in another community

$$\tau(C) = \frac{(\#\text{ of community triangles in } C)}{(\#\text{ of cut triangles in } C) + (\text{overlap size}) + |C|}$$

Theorem. The overlapping community detection problem is NP-hard.

Overlapping community detection algorithm

Algorithm 4 OverlappingCommunityDetection(G)

$\mathcal{C}_0 \leftarrow \{V\}$; $k \leftarrow 0$;
/* Find core communities */
while there is an edge with support no larger than k **do**
    for each community $C \in \mathcal{C}_k$ **do**
        Update C in $\mathcal{C}_{k+1}$ if removal of edges increases τ;
    $k \leftarrow k + 1$;
/* Expand core communities in $\mathcal{C}_k$ */
for each core $C \in \mathcal{C}$ **do**
    for each neighbor u of vertex v **do**
        if $\tau(C \cup \{u\}) \geq \tau(C)$ **then**
            Add the neighbor u to C;
return $\mathcal{C}$;

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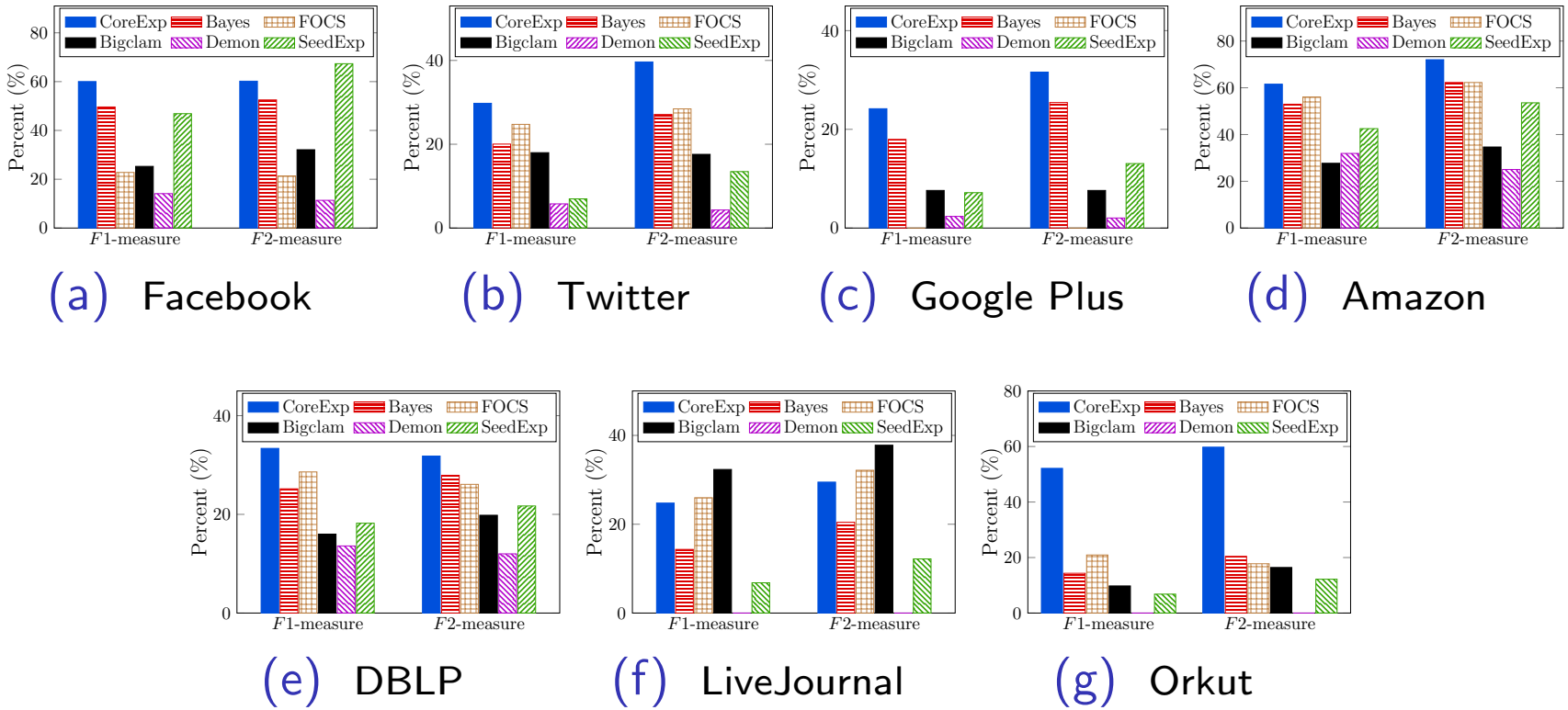
Empirical evaluations



(a) Facebook (b) Twitter (c) Google Plus (d) Amazon

(e) DBLP (f) LiveJournal (g) Orkut

Figure 9 : The quality of overlapping communities found.

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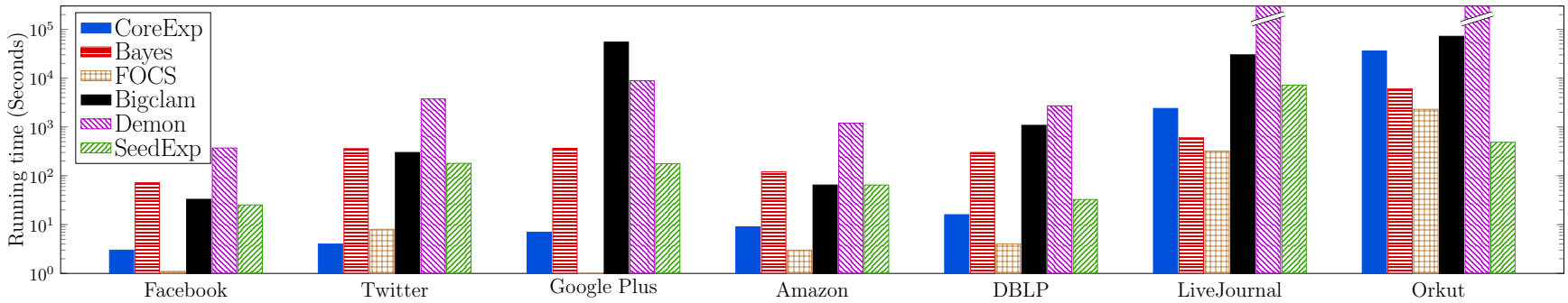
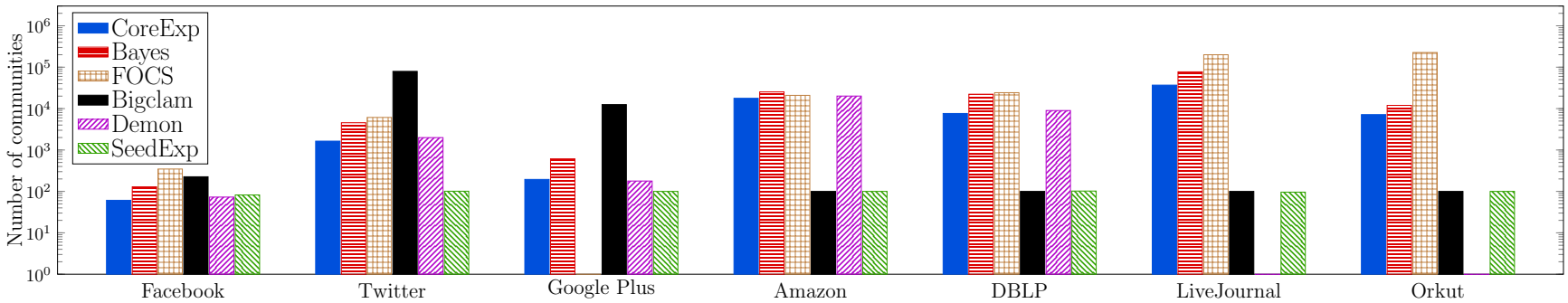
Empirical evaluations



(a) Running times of different algorithms



(b) Number of communities found

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Community search vs. community detection

- Given a set of query vertices, we are interested in finding communities of those vertices.
- In community detection, we are interested in finding all communities of a network.

Existing approaches

Find only one community that includes the query vertices:

- k-core
- k-truss
- k-truss with minimum diameter
- γ-quasi k-cliques
- γ-quasi k-cliques with k triangles

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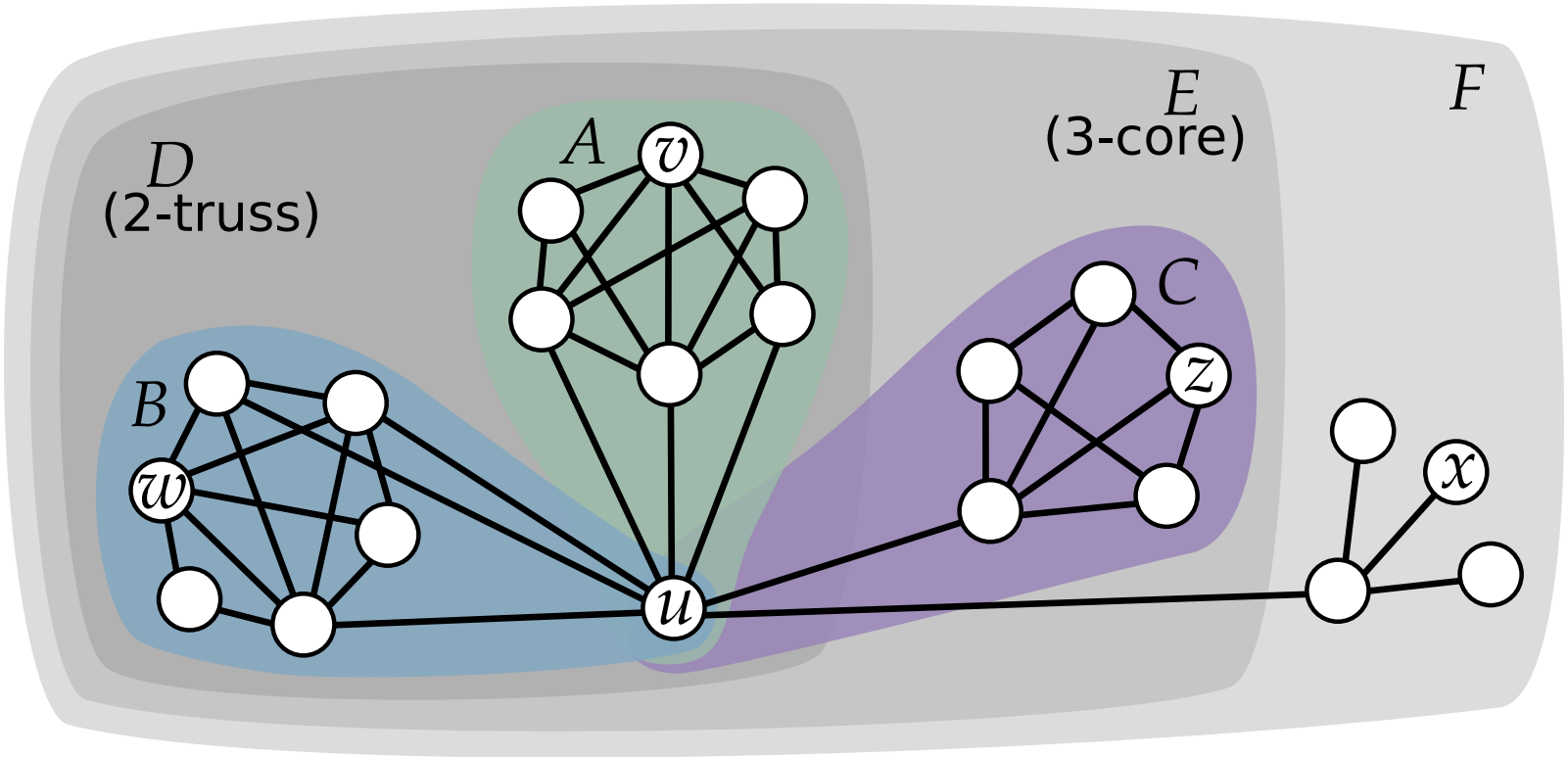
Illustrative example

Query vertices may belong to different communities:



Propinquity

The propinquity value between vertices u and v, i.e. $\Delta(u, v)$, represents the closeness between them.

Propinquity measures can have properties such as,

- Reflexive
- Symmetric
- Transitive (with respect to a value k)

Community search problem

Find a minimum cover $\mathcal{C} = \{V_1, \cdots, V_t\}$ $(t \leq |Q|)$ such that:

- the propinquity between every two vertices u and v in V_i $(V_i \in \mathcal{C})$ is no less than k, i.e. $\Delta(V_i) \geq k$,
- every community $V_i \in \mathcal{C}$ is maximal, and
- for every pair of query vertices $q_i, q_j \in Q$,

$$\Delta(q_i, q_j) \geq k \Rightarrow \exists\ V_p \in \mathcal{C}\ \ (\{q_i, q_j\} \subseteq V_p).$$

Theorem. The problem of identifying top-k structural hole spanners is NP-hard.

A novel propinquity measure

- A small length of shortest path between vertices denotes the closeness between them
- Hubs may obscure the accuracy of shortest paths
- The sum of top-k shortest paths between two vertices denotes propinquity between two vertices

The number of top-k shortest paths between u and v whose sum is no larger than ℓ.

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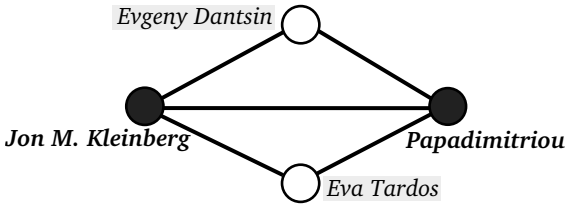
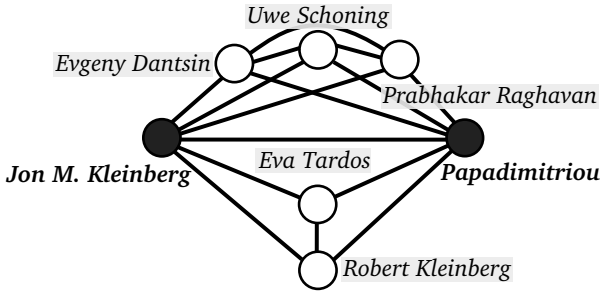
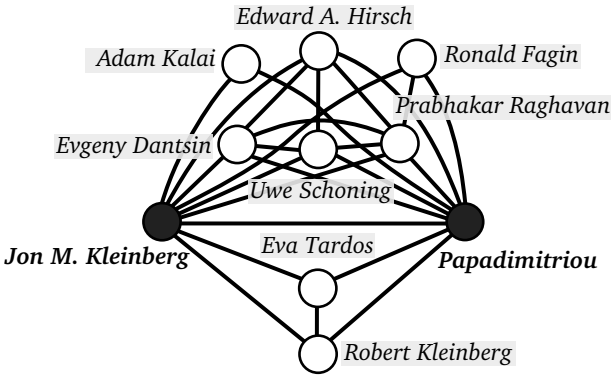
An illustrative example



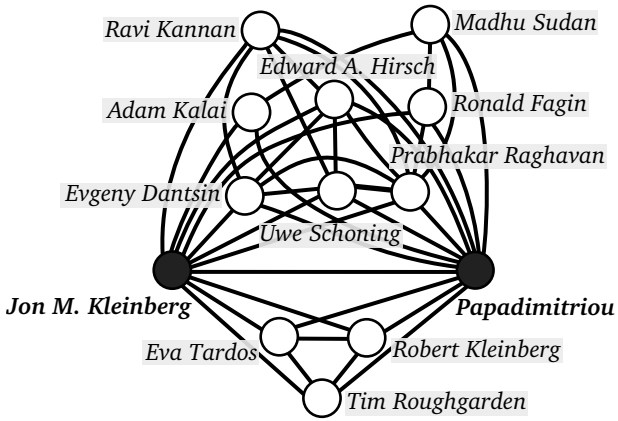
(c) $\ell = 5$



(d) $\ell = 11$



(e) $\ell = 17$



(f) $\ell = 23$

Clique-based algorithm for community search

1. Find the *query relevance graph* (QRG);
2. Find **cliques** of QRG;
3. For each **clique** in QRG:
4. Discover the top-k shortest paths between query vertices in the clique;
5. Return the expanded communities;

Fast algorithm for community search

1. Find the *query relevance graph* (QRG);
2. Find **connected components** of QRG;
3. For each **connected component** in QRG:
4. Discover the top-k shortest paths between query vertices in the clique;
5. Return the expanded communities;

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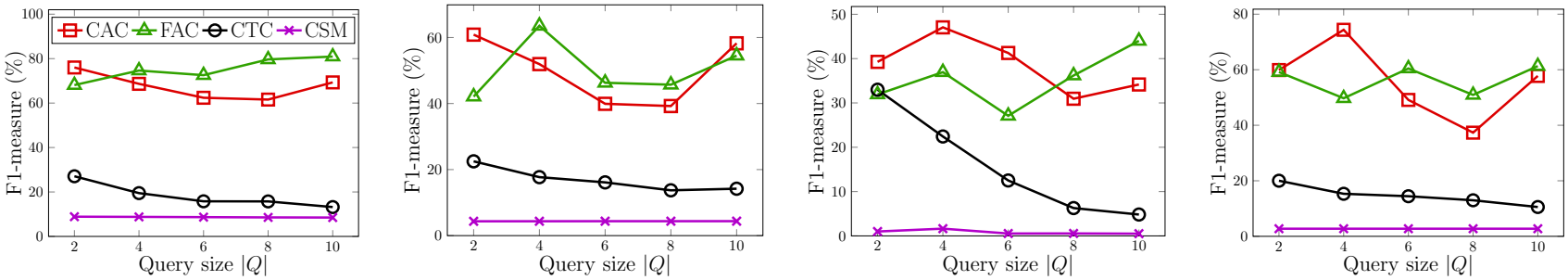
Empirical evaluations



(g) Amazon (h) DBLP (i) Youtube (j) LiveJournal

(k) Amazon (l) DBLP (m) Youtube (n) LiveJournal

Figure 10 : F1-measure and F2-measure of different algorithms on random queries of various sizes.

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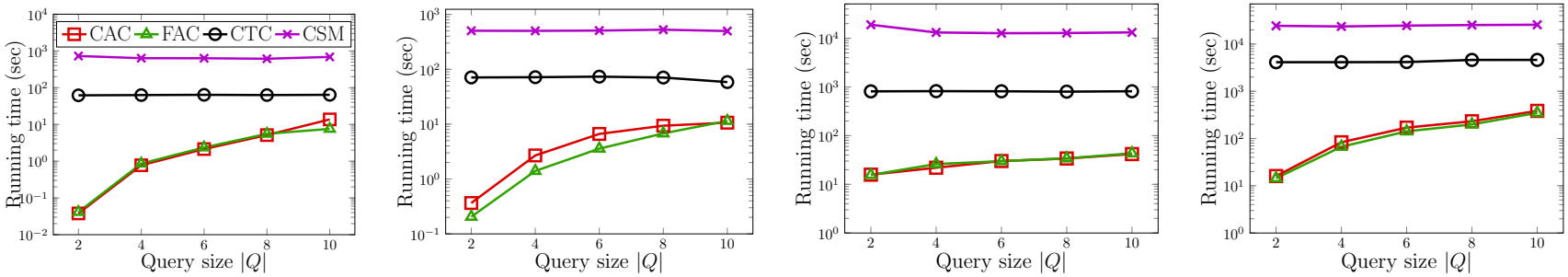
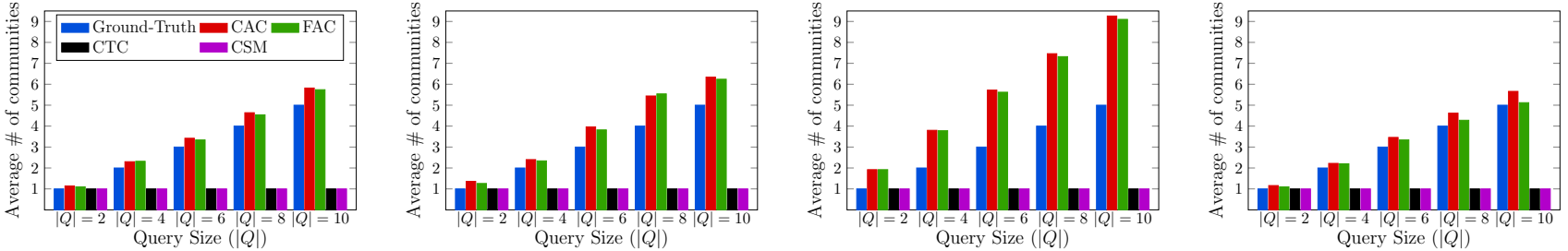
Empirical evaluations



(a) Amazon (b) DBLP (c) Youtube (d) LiveJournal

Figure 11 : Running time of different algorithms on random queries.



(a) Amazon (b) DBLP (c) Youtube (d) LiveJournal

Figure 12 : Average number of communities detected for each queries.

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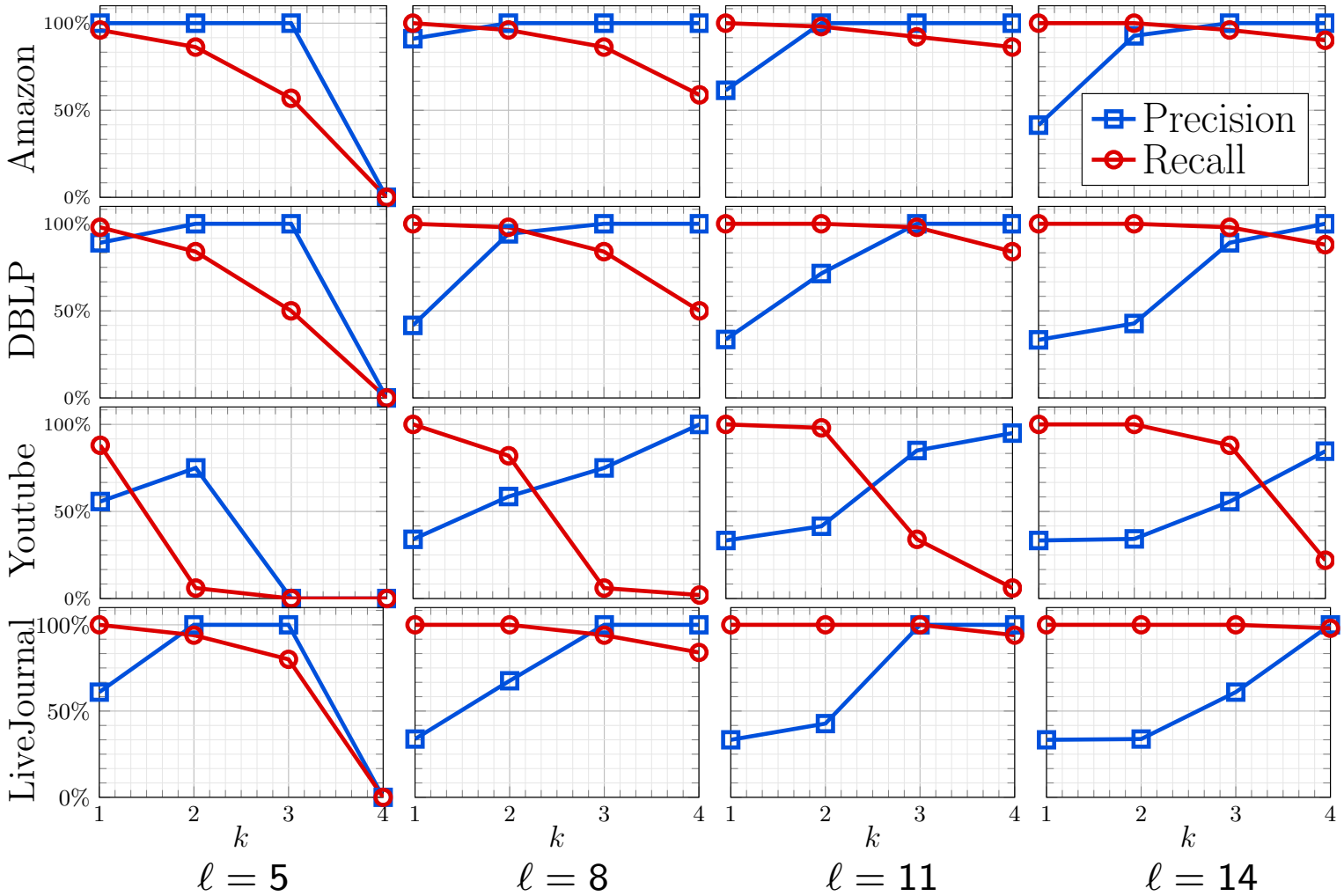
Parameter evaluations



Outline

- Community structure in complex networks
- Hierarchical community structure
- Structural hole spanners
- Overlapping community structure
- Community search
- Conclusion

Conclusion & future works

Conclusion

- The thesis formally defined each problem
- The thesis devised scalable algorithms for them
- The thesis evaluated the performance of the proposed algorithms by comparing against benchmark algorithms

Future works

- Extension to weighted and directed graphs
- Using different edge and vertex weighted methods
- Extension to dynamically evolving networks

Thank You