# DK-PERSONALIZATION: PUBLISHING NETWORK STATISTICS WITH PERSON-ALIZED DIFFERENTIAL PRIVACY

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- Experiments and Results



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- Conclusion and Future Work

## INTRODUCTION



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- However, given the private nature of data about individuals stored in networks, releasing network data raises privacy concerns, thereby requiring privacy-preserving mechanisms.
- The current focus of privacy is around differential privacy (DP). However, a uniform privacy level (i.e., ε<sup>1</sup>) is assigned to each individual while guaranteeing DP, which may over or under protect individuals.

<sup>&</sup>lt;sup>1</sup>Smaller value of  $\varepsilon$  implies a stronger privacy guarantee.





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  - Each individual (node) has its own privacy preference whereas each data point in data distribution reflects information about more than one node.



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### Key Challenges:

- Each individual (node) has its own privacy preference whereas each data point in data distribution reflects information about more than one node.
- Network data is highly sensitive to structural changes under DP.

## **PROBLEM FORMULATION**



We define the notion of neighboring graphs ( $G \sim G'$ ) under edge and node-DP.

## **NEIGHBORING GRAPHS**



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Original-Graph



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 γ<sup>dK</sup>(G) queries the dK-distribution of G.



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- Given two (edge or node) neighboring graphs *G* ~ *G*′ where *G*′ is obtained from *G* by adding (or deleting) an edge (or node) can affect more than one node.
- Thus, PDP should be formalized in terms of all affected nodes to guarantee *ε*-indistinguishability.
- Given a privacy specification  $\Phi = \{\varepsilon_1, \dots, \varepsilon_n\}$ , denote  $\Phi^v$  the privacy preference  $\varepsilon$  of a node v



**Edge**  $\Phi$ -**PDP:** For **G**  $\stackrel{e}{\sim}$  **G**', adding (or deleting) an edge affects exactly two nodes *u* and *v*.

$$\Pr[\mathcal{K}(G) \in \mathcal{O}] \leq e^{\min\{\Phi^u, \Phi^v\}} \times \Pr[\mathcal{K}(G') \in \mathcal{O}].$$



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**Node**  $\Phi$ -**PDP:** For **G**  $\stackrel{n}{\sim}$  **G**', adding (or deleting) a node  $v^+$  affects  $|E^+|$  nodes incident to  $v^+$  and  $v^+$  itself.

$$\Pr[\mathcal{K}(G) \in \mathcal{O}] \le e^{\min\{\Phi^{v} | (v^+, v) \in E^+\}} imes \Pr[\mathcal{K}(G') \in \mathcal{O}]$$
 and  
 $\Pr[\mathcal{K}(G) \in \mathcal{O}] \le e^{\Phi^{v^+}} imes \Pr[\mathcal{K}(G') \in \mathcal{O}]$ 



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### (DEGREE QUERY)

A degree query  $\gamma_q : \gamma^{d\mathcal{K}}(G) \to \mathbb{N}$  maps a degree tuple  $d_t \in \gamma^{d\mathcal{K}}(G)$ to a frequency value in  $\mathbb{N}$  s.t.  $(d_t, \gamma_q(G)) \in \gamma^{d\mathcal{K}}(G)$ .

## SENSITIVITY ANALYSIS


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We observe that the sensitivity of  $\gamma_q$  is half as compared to  $\gamma^{dK}$ .

<sup>&</sup>lt;sup>2</sup>The maximum change in  $\gamma_q$ .

#### PROPOSED PERSONALIZED APPROACHES



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The frequency value 2 in 1*K*(*G*), and the frequency value 3 in 2*K*(*G*) are perturbed with  $\varepsilon = min(\Phi^B, \Phi^F)$ , and  $\varepsilon = min(\Phi^A, \Phi^C, \Phi^D, \Phi^E)$ , respectively.























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With threshold  $\tau$  all nodes with high privacy are removed.























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Sampled dK-distribution is perturbed with  $\tau$ .

























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```
\gamma_{q} is approximated to \gamma_{q} \circ \mathcal{M}.
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#### **EXPERIMENTS AND RESULTS**



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#### **EXPERIMENTAL SETUP**



#### Four network datasets:

- (1) *Facebook* contains 4,039 nodes and 88,234 edges.
- (2) Wiki-Vote contains 7,115 nodes and 103,689 edges.
- (3) *Ca-HepPh* contains 12,008 nodes and 118,521 edges.
- (4) Email-Enron contains 36,692 nodes and 183,831 edges.
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- Two utility metrics [1]:
  - ► L1 distance measures the network structural error the original dK-distribution D and its private version  $D_{\Phi}$  by calculating  $\|D D_{\Phi}\|_1 = \sum_{i=1}^{\deg(G)} |D_i D_{\Phi_i}|.$



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  - ► KS distance measures the closeness between the cumulative distribution functions of D and  $D_{\Phi}$  by calculating  $KS(D, D_{\Phi}) = max_i |CDF_{D_i} CDF_{D_{\Phi_i}}|$ .











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- AG-dK outperforms under edge-PDP and LL-dK outperforms under node-PDP by generating more similar 2K-distributions.





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- Increasing  $\varepsilon$  and decreasing  $\Delta$  can help to reduce error.
- Reducing sensitivity is more challenging under node-PDP than for edge-PDP as graph data is highly sensitive under node-DP.

# **CONCLUSION AND FUTURE WORK**





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- We have proposed four personalized privacy-preserving mechanisms while enhancing output utility.
- The effectiveness of our proposed work has been empirically verified over four real-world networks.
- **Future work:** To this work will consider local differential privacy to release network statistics under personalization.

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# **THANKS FOR YOUR ATTENTION!**

# ANY QUESTIONS