BatchHL: Answering Distance Queries on Batch-Dynamic Networks at Scale

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- **Problem:** Let $G \hookrightarrow G'$ denote that G is changed to G' by a *batch update*, to answer the shortest path distance $d_{G'}(s, t)$.

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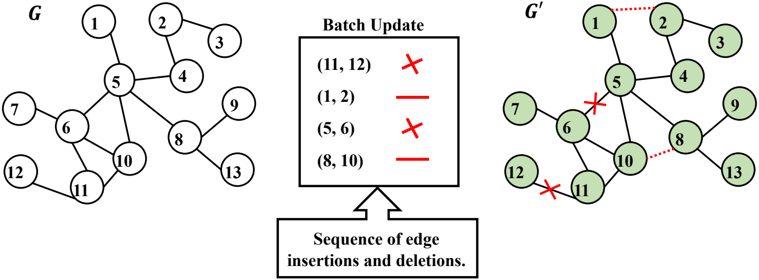

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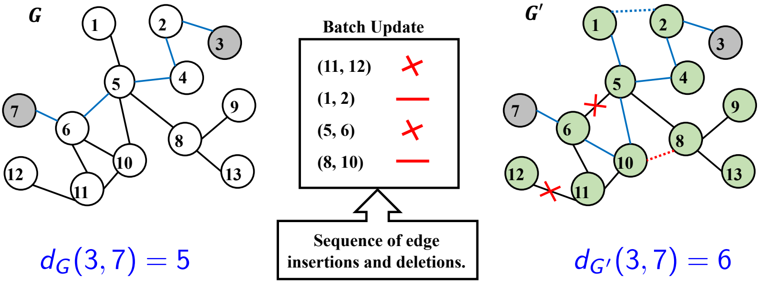


$d_G(3, 7) = 5$

$d_{G'}(3, 7) = 6$

Applications

World Wide Web Networks

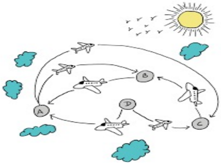
Context-Aware Web Search

Social Networks

Socially-Sensitive Search

Social Network Analysis
centrality, community search

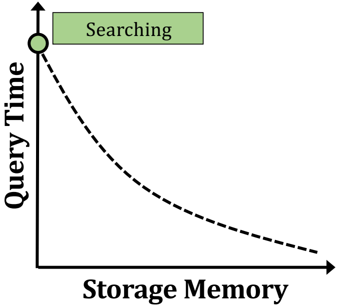
Road Networks



Route Navigation

⋮

Existing Approaches



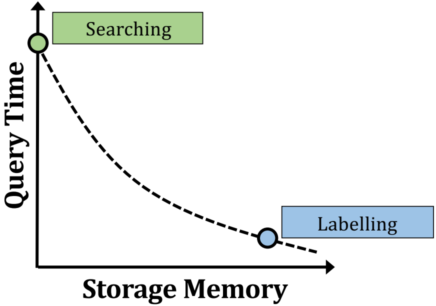
Dynamic Methods

Search-based Methods

- Slow querying

Labelling-based Methods

- Decremental Method (DECM) (Qin+, WWW'17)
- Dynamic 2-Hop Cover (FulPLL) (D'angelo+, JEA'19)
- Maintenance of 2-Hop cover (DWPSL) (Zhang+, ICDE'21)



Existing Approaches

Dynamic Methods

Search-based Methods

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Decremental Method (DECM)

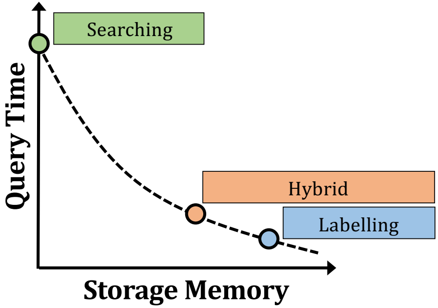
- Limited Scalability
- Slow update time
- No minimality for Incremental updates

(Zhang+, ICDE'21)

Hybrid Methods

Fully Dynamic (FD)
(Hayashi+, CIKM'16)

Maintenance of Highway cover (HL)
(Farhan+, JVLDB'2021)



Existing Approaches

Dynamic Methods

Search-based Methods

- Slow querying

Labelling-based Methods

Decremental Method (DECM)

- Limited Scalability
- Slow update time
- No minimality for Incremental updates

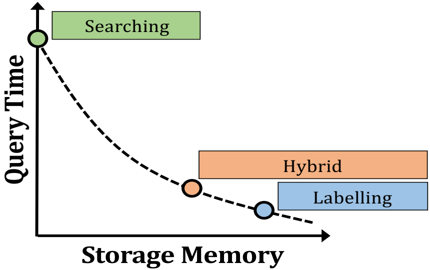
(Zhang+, ICDE'21)

Hybrid Methods

- process single update at a time
- Slow update time

(Farhan+, JVLDB'2021)

Batch-Dynamic Highway Cover Labelling
(Our Work)



- **Distance Labelling:**

  Given a set of landmarks $R \subseteq V$, precompute a label $L(v)$ for every vertex $v \in V$ in G.

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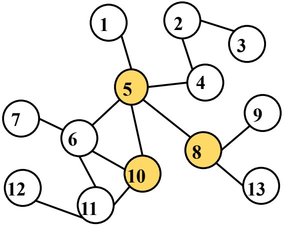

$R = \{5, 8, 10\}$

$L(1)$

Landmark	5	8	10
Distance	1	2	2

$L(2)$

Landmark	5	8	10
Distance	2	3	3

$d_G(8, 2) = 3$

$L(3)$

Landmark	5	8	10
Distance	3	4	4

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- **2-Hop Cover Labelling:**

  Distance through a common landmark in $L(s)$ and $L(t)$.

$$d_G(s,t) = \min_{r \in L(s) \cap L(t)} \{\delta(r,s) + \delta(r,t)\}$$

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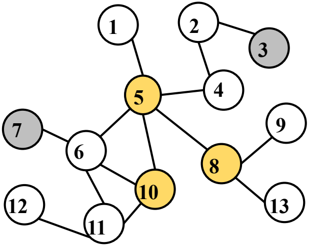


$R = \{5, 8, 10\}$

$L(3)$

Landmark	5
Distance	3

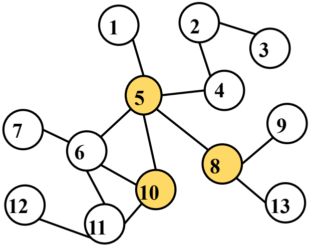
$L(7)$

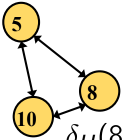
Landmark	5	10
Distance	2	2

$d_G(3,7) = d_G(3,5) + d_G(7,5) = 5$

Definition (Highway)

A highway H is a pair (R, δ_H), where R is a set of landmarks and δ_H is a *distance decoding function*, i.e. $\delta_H : R \times R \rightarrow \mathbb{N}^+$, such that for any $\{r_1, r_2\} \subseteq R$ we have $\delta_H(r_1, r_2) = d_G(r_1, r_2)$.

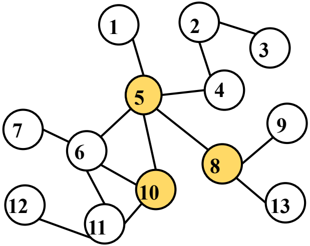




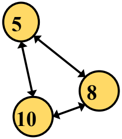
$\delta_H(8, 10) = 2$

Definition (Highway Cover)

Let $G = (V, E)$ be a graph and $H = (R, \delta_H)$ a highway. For any vertex $u \in V \backslash R$ and any $r \in R$, there must exist $r' \in R$ in $L(u)$ such that r' is on a shortest path between u and r (r and r' may be the same).



Graph



Highway

Label	Distance Entries
$L(1)$	(5, 1)
$L(2)$	(5, 2)
$L(3)$	(5, 3)
$L(4)$	(5, 1)
$L(6)$	(5, 1) (10, 1)
$L(7)$	(5, 2) (10, 2)
$L(9)$	(8, 1)
$L(11)$	(10, 1)
$L(12)$	(10, 1)
$L(13)$	(8, 1)

Labelling

Two steps for answering $d_G(s, t)$ in a graph G:

(1) Computing an upper bound distance using the precomputed (offline) highway cover labelling;

(2) Computing $d_G(s, t)$ using the (online) bidirectional search over a sparsified graph $G[V \backslash R]$.

- Find a path of the minimal length through a highway H:

$$d_{st}^{\top} = \min_{\substack{(r_i,\delta(r_i,s))\in L(s) \\ (r_j,\delta(r_j,t))\in L(t)}} \{\delta(r_i, s) + \delta_H(r_i, r_j) + \delta(r_j, t)\}$$

- Find a path of the minimal length through a highway H:

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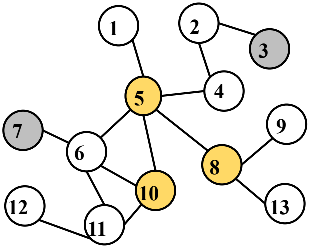


What is $d^{\top}_{st}$ for $s = 3$ and $t = 7$?

$L(7)$

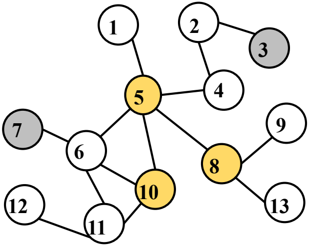
Landmark	5	10
Distance	2	2

$L(3)$

Landmark	5
Distance	3

- Find a path of the minimal length through a highway H:

$$d^{\top}_{st} = \min_{\substack{(r_i,\delta(r_i,s))\in L(s)\\(r_j,\delta(r_j,t))\in L(t)}} \{\delta(r_i,s) + \delta_H(r_i,r_j) + \delta(r_j,t)\}$$



What is $d^{\top}_{st}$ for $s = 3$ and $t = 7$?

$L(7)$

Landmark	5	10
Distance	2	2

$L(3)$

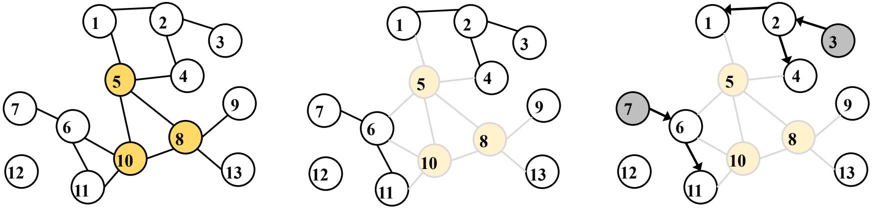
Landmark	5
Distance	3

$3 \to 5 \to 7$: length 5

$3 \to 5 \to 10 \to 7$: length 6

$d^{\top}_{st} = 5$

- Sparsify graph G by removing all landmarks in R, i.e. $G' = G[V \backslash R]$
- Conduct a bidirectional search on G' which is bounded by $d_{st}^{\top} - 1$



$$d_{G'}(3,7) = 5$$

- **Aim:**

  Efficient maintenance of highway cover distance labelling for a very large dynamic network undergoing batches of updates?

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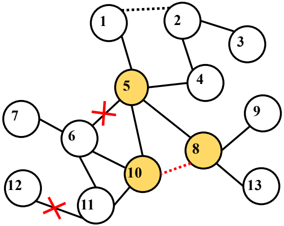
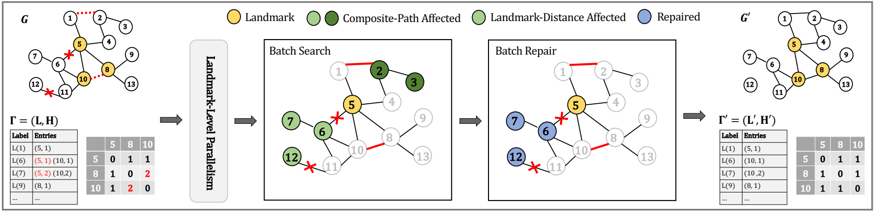


Changed Graph G'

Label	Entries
L(1)	(5, 1)
L(6)	(5, 1) (10, 1)
L(7)	(5, 2) (10,2)
L(9)	(8, 1)
...	...

	5	8	10
5	0	1	1
8	1	0	2
10	1	2	0

Highway Labelling



G
Landmark
Composite-Path Affected
Landmark-Distance Affected
Repaired
G′
Γ = (L, H)
Label | Entries
L(1) | (5, 1)
L(6) | (5, 1) (10, 1)
L(7) | (5, 2) (10,2)
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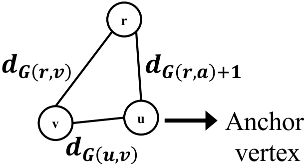
... | ...
| 5 | 8 | 10
5 | 0 | 1 | 1
8 | 1 | 0 | 2
10 | 1 | 2 | 0
Landmark-Level Parallelism
Batch Search
Batch Repair
Γ′ = (L′, H′)
Label | Entries
L(1) | (5, 1)
L(6) | (10, 1)
L(7) | (10 ,2)
L(9) | (8, 1)
... | ...
| 5 | 8 | 10
5 | 0 | 1 | 1
8 | 1 | 0 | 1
10 | 1 | 1 | 0

Batch Search

- Unify an edge insertion and deletion (a, b) into a single process
  - both share the same pattern for affected vertices



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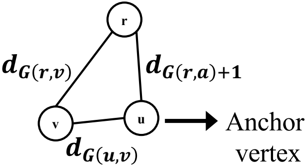
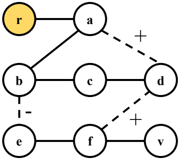


- Searches for vertices affected by edge insertions and deletions in a batch combine in a single search



(+) Inserted, (-) Deleted

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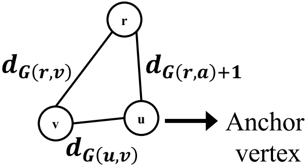

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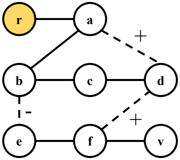

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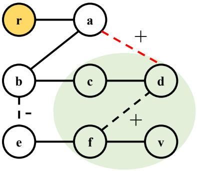

Insertion of (a, d)

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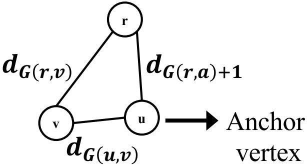
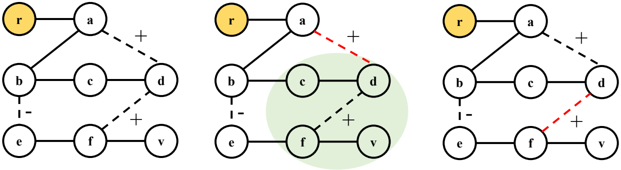


- Searches for vertices affected by edge insertions and deletions in a batch combine in a single search



(+) Inserted, (-) Deleted   Insertion of (a, d)   Insertion of (d, f)

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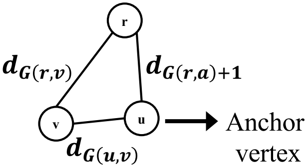

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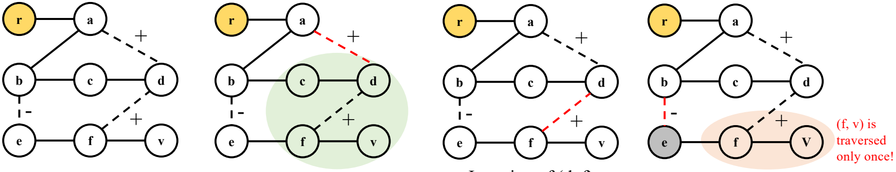

(+) Inserted, (-) Deleted   Insertion of (a, d)   Insertion of (d, f)   Deletion of (b, e)

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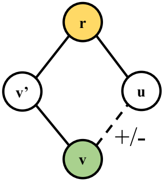
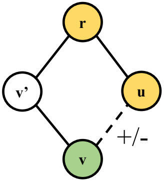
- New shortest path of the same length as existing ones won't change the distance



Case 1: no change



Case 2: change

Improved Batch Search

- To identify such cases, we track whether a shortest path to r passes through (1) another landmark and (2) a deleted edge

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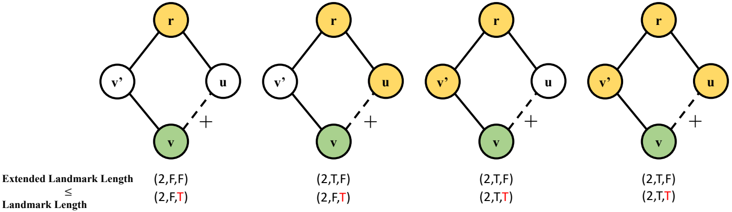

Addition Case - Pruning

Improved Batch Search

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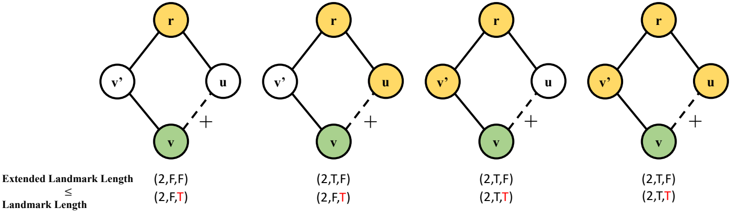
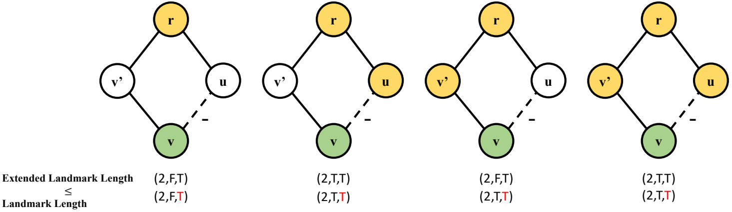


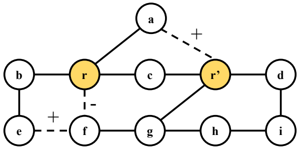
Addition Case - Pruning



Deletion Case - Pruning

Batch Repair

- Repair the labels of vertices returned by batch search



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1)		(r, 2)	(r, 1)	(r, 2)	(r, 3)	
		(r',1)	(r', 1)		(r', 2)	(r', 1)	(r', 2)	(r', 2)

Batch Repair

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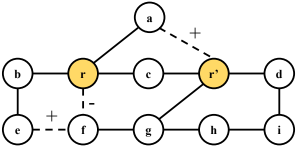
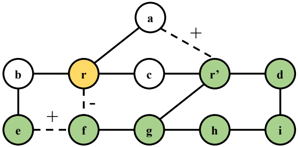


$H = \{\delta_H(\mathrm{r}, \mathrm{r'}) = 2\}$

L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 1) (r', 2)	(r, 2) (r', 1)	(r, 3) (r', 2)	(r', 2)



Basic batch search algorithm
returns {r', d, e, f, g, h, i}

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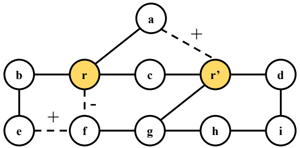
- Repair the labels of vertices returned by batch search



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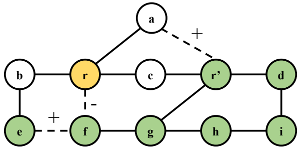

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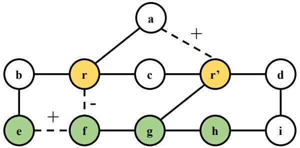
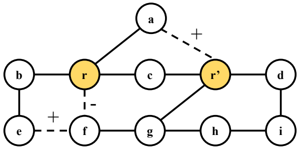


Improved batch search algorithm returns {e, f, g, h}

Batch Repair

- Repair the labels of vertices returned by batch search



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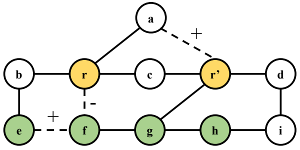
L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1)		(r, 2)	(r, 1)	(r, 2)	(r, 3)	
		(r',1)	(r', 1)		(r', 2)	(r', 1)	(r', 2)	(r', 2)



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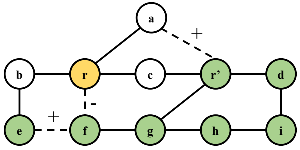
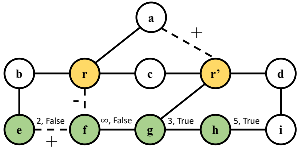
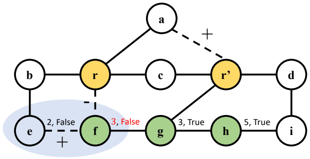


Basic batch search algorithm returns {r', d, e, f, g, h, i}



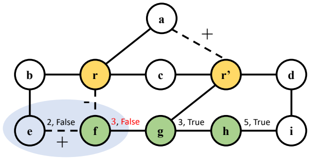


$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r’,1)	(r’, 1)	(r, 2)	(r, 1) (r’, 2)	(r, 2) (r’, 1)	(r, 3) (r’, 2)	(r’, 2)

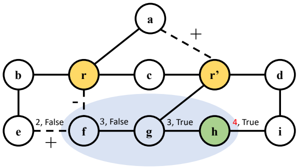
Batch Repair



$H = \{\delta_H(\mathrm{r}, \mathrm{r'}) = 2\}$

L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 1) (r', 2)	(r, 2) (r', 1)	(r, 3) (r', 2)	(r', 2)

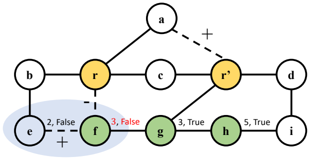


$H = \{\delta_H(\mathrm{r}, \mathrm{r'}) = 2\}$

L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 3) (r', 2)	(r, 2) (r', 1)	(r, 3) (r', 2)	(r', 2)

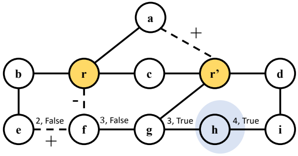
Batch Repair



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

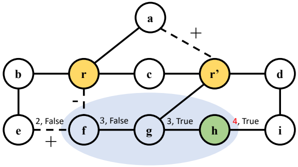
a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 1) (r', 2)	(r, 2) (r', 1)	(r, 3) (r', 2)	(r', 2)



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

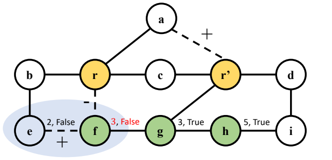
a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 3) (r', 2)	(r', 1)	~~(r, 3)~~ (r', 2)	(r', 2)



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

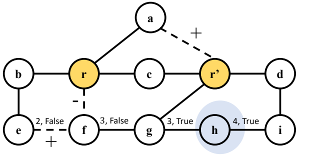
a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 3) (r', 2)	(r, 2) (r', 1)	(r, 3) (r', 2)	(r', 2)



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

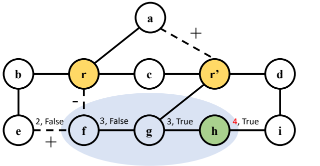
a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 1) (r', 2)	(r, 2) (r', 1)	(r, 3) (r', 2)	(r', 2)



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

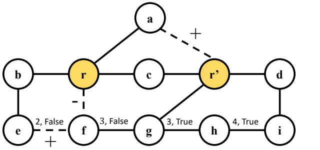
a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 3) (r', 2)	(r', 1)	~~(r, 3)~~ (r', 2)	(r', 2)



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 3) (r', 2)	(r, 2) (r', 1)	(r, 3) (r', 2)	(r', 2)



$H = \{\delta_H(\mathrm{r}, \mathrm{r}') = 2\}$

L =

a	b	c	d	e	f	g	h	i
(r, 1)	(r, 1)	(r, 1) (r',1)	(r', 1)	(r, 2)	(r, 3) (r', 2)	(r', 1)	(r', 2)	(r', 2)

Empirical Evaluation

Dataset	$\|V\|$	$\|E\|$	Fully Dynamic Batch Update Time (sec.)						Incremental Batch Update Time (sec.)					Decremental Batch Update Time (sec.)				
			BHL^p	BHL$^+$	BHL	UHL$^+$	FulFD	FulPLL	BHL^p	BHL$^+$	UHL$^+$	IncFD	IncPLL	BHL^p	BHL$^+$	UHL$^+$	DecFD	DecPLL
Youtube	1.1M	3M	**0.046**	**0.070**	**0.208**	0.091	1.249	9.110	**0.003**	**0.008**	0.048	0.154	0.194	**0.070**	**0.169**	0.239	3.181	9.850
Skitter	1.7M	11M	**0.147**	**0.601**	**0.902**	1.587	5.986	8.770	**0.002**	**0.006**	0.069	0.117	1.312	**0.163**	**0.751**	2.382	14.15	31.50
Flickr	1.7M	16M	**0.024**	**0.026**	**0.130**	0.099	2.152	6.300	**0.003**	**0.008**	0.072	0.053	1.259	**0.030**	**0.041**	0.107	3.364	13.40
Wikitalk	2.4M	5M	**0.029**	**0.025**	**0.101**	0.134	2.926	4.550	**0.002**	**0.005**	0.097	0.029	0.081	**0.046**	**0.044**	0.147	5.674	9.820
Hollywood	1.1M	114M	**0.008**	**0.014**	**0.115**	0.056	4.423	-	**0.001**	**0.002**	0.046	0.090	27.53	**0.017**	**0.031**	0.071	8.401	-
Orkut	3.1M	117M	**0.537**	**1.775**	**5.855**	4.539	13.30	-	**0.005**	**0.014**	0.127	0.367	-	**0.677**	**0.035**	5.921	23.94	-
Enwiki	4.3M	101M	**0.508**	**1.681**	**10.50**	3.952	121.7	-	**0.008**	**0.012**	0.168	0.316	4.916	**0.770**	**3.079**	8.194	251.2	-
Livejournal	4.8M	68M	**0.221**	**0.306**	**0.873**	0.379	4.736	-	**0.006**	**0.010**	0.202	0.244	-	**0.299**	**0.570**	0.731	4.736	-
Indochina	7.4M	194M	**0.543**	**1.181**	**1.547**	9.575	20.63	-	**0.015**	**0.011**	0.308	0.141	4.680	**0.553**	**1.346**	19.20	44.92	-
Twitter	42M	1.5B	**13.29**	**49.62**	**115.7**	125.6	5103	-	**0.125**	**0.024**	13.09	0.263	-	**19.17**	**68.85**	231.8	9460	-
Friendster	66M	1.8B	**0.409**	**0.410**	**0.811**	21.93	23.27	-	**0.163**	**0.035**	20.96	0.254	-	**0.420**	**0.738**	21.87	30.38	-
UK	106M	3.7B	**14.45**	**41.46**	**40.79**	56.50	110.1	-	**0.218**	**0.055**	4.349	0.258	-	**14.99**	**42.29**	75.20	257.3	-
Italianwiki	1.2M	35M	**0.001**	**0.001**	**0.025**	0.051	6.623	-	-	-	-	-	-	-	-	-	-	-
Frenchwiki	2.2M	59M	**0.003**	**0.004**	**0.067**	0.098	5.289	-	-	-	-	-	-	-	-	-	-	-

Empirical Evaluation

Dataset	$\|V\|$	$\|E\|$	Fully Dynamic Batch Update Time (sec.)						Incremental Batch Update Time (sec.)					Decremental Batch Update Time (sec.)				
			BHL^p	BHL$^+$	BHL	UHL$^+$	FulFD	FulPLL	BHL^p	BHL$^+$	UHL$^+$	IncFD	IncPLL	BHL^p	BHL$^+$	UHL$^+$	DecFD	DecPLL
Youtube	1.1M	3M	**0.046**	**0.070**	**0.208**	0.091	1.249	9.110	**0.003**	**0.008**	0.048	0.154	0.194	**0.070**	**0.169**	0.239	3.181	9.850
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Livejournal	4.8M	68M	**0.221**	**0.306**	**0.873**	0.379	4.736	-	**0.006**	**0.010**	0.202	0.244	-	**0.299**	**0.570**	0.731	4.736	-
Indochina	7.4M	194M	**0.543**	**1.181**	**1.547**	9.575	20.63	-	**0.015**	**0.011**	0.308	0.141	4.680	**0.553**	**1.346**	19.20	44.92	-
Twitter	42M	1.5B	**13.29**	**49.62**	**115.7**	125.6	5103	-	**0.125**	**0.024**	13.09	0.263	-	**19.17**	**68.85**	231.8	9460	-
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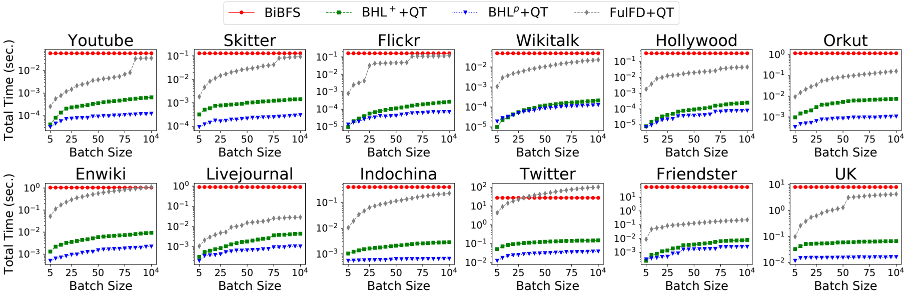
Frenchwiki	2.2M	59M	**0.003**	**0.004**	**0.067**	0.098	5.289	-	-	-	-	-	-	-	-	-	-	-

- Much improved update time for all three settings.
- Decremental batch updates are much more faster.

Empirical Evaluation



Comparison of the total time of querying and updating the labelling with the baseline methods.

Conclusion

- Contributions

  An efficient batch update method for answering distance queries on graphs undergoing batch updates, which has the following benefits

  (1) Unifying edge insertion and deletion

  (2) Avoiding unnecessary and repeated computations

  (3) Exploiting the potential for parallelism

- Future works
  - Extension of the proposed approaches to road networks
  - Selection of highly central vertices
  - Guided search by investigating properties of the proposed approaches

Questions

Thank You